Linear Programming Problems

A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.

The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

- Formulate the problem of deciding how much of each product to make in the current week as a linear program.
- Solve this linear program graphically.

Solution

Let

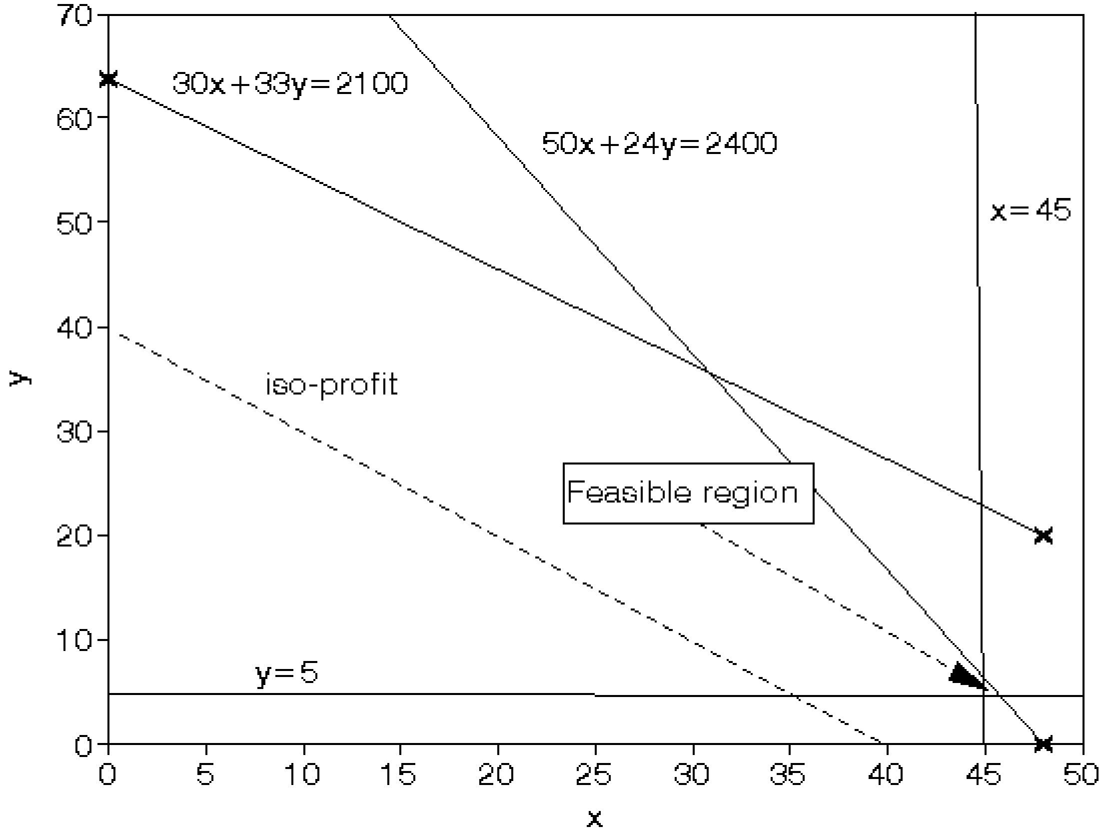
- x be the number of units of X produced in the current week
- y be the number of units of Y produced in the current week

then the constraints are:

- $50x + 24y <= 40(60)$ machine A time
- $30x + 33y <= 35(60)$ machine B time
- $x >= 75 - 30$
- i.e. $x >= 45$ so production of X >= demand (75) - initial stock (30), which ensures we meet demand
- $y >= 95 - 90$
- i.e. $y >= 5$ so production of Y >= demand (95) - initial stock (90), which ensures we meet demand

The objective is: maximise $(x+30-75) + (y+90-95) = (x+y-50)$
i.e. to maximise the number of units left in stock at the end of the week

It is plain from the diagram below that the maximum occurs at the intersection of $x=45$ and $50x + 24y = 2400$



Solving simultaneously, rather than by reading values off the graph, we have that x=45 and y=6.25 with the value of the objective function being 1.25

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2) A company is involved in the production of two items (X and Y). The resources need to produce X and Y are twofold, namely machine time for automatic processing and craftsman time for hand finishing. The table below gives the number of minutes required for each item:

		Machine time	Craftsman time
Item	X	13	20
	Y	19	29

The company has 40 hours of machine time available in the next working week but only 35 hours of craftsman time. Machine time is costed at £10 per hour worked and craftsman time is costed at £2 per hour worked. Both machine and craftsman idle times incur no costs. The revenue received for each item produced (all production is sold) is £20 for X and £30 for Y. The company has a specific contract to produce 10 items of X per week for a particular customer.

- Formulate the problem of deciding how much to produce per week as a linear program.
- Solve this linear program graphically.

Solution

Let

- x be the number of items of X
- y be the number of items of Y

then the LP is:

maximise

- 20x + 30y - 10(machine time worked) - 2(craftsman time worked)

subject to:

- 13x + 19y <= 40(60) machine time
- 20x + 29y <= 35(60) craftsman time
- x >= 10 contract
- x,y >= 0

so that the objective function becomes

maximise

- 20x + 30y - 10(13x + 19y)/60 - 2(20x + 29y)/60

i.e. maximise

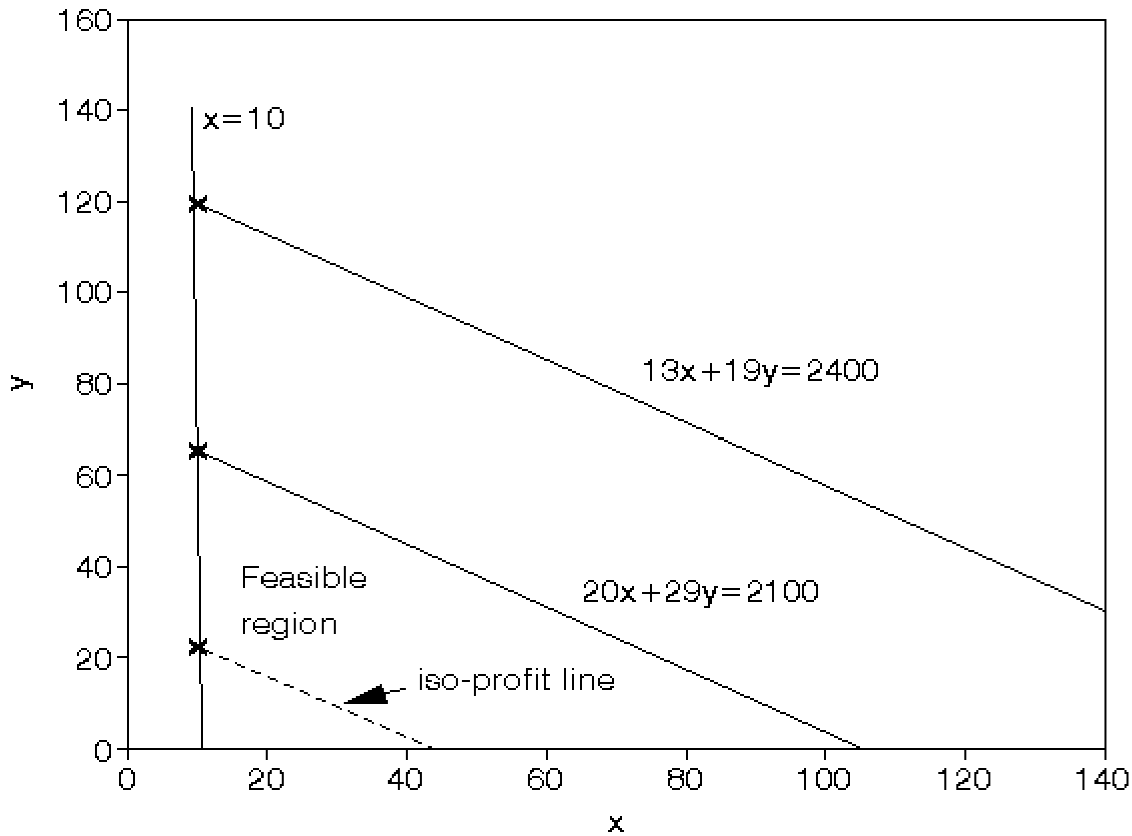
- 17.1667x + 25.8667y

subject to:

- 13x + 19y <= 2400
- 20x + 29y <= 2100
- x >= 10
- x,y >= 0

It is plain from the diagram below that the maximum occurs at the intersection of x=10 and 20x + 29y <= 2100

Solving simultaneously, rather than by reading values off the graph, we have that x=10 and y=65.52 with the value of the objective function being £1866.5



Linear programming example 1992 UG exam

A company manufactures two products (A and B) and the profit per unit sold is £3 and £5 respectively. Each product has to be assembled on a particular machine, each unit of product A taking 12 minutes of assembly time and each unit of product B 25 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 30 hours (due to maintenance/breakdown).

Technological constraints mean that for every five units of product A produced at least two units of product B must be produced.

- Formulate the problem of how much of each product to produce as a linear program.
- Solve this linear program graphically.
- The company has been offered the chance to hire an extra machine, thereby doubling the effective assembly time available. What is the *maximum* amount you would be prepared to pay (per week) for the hire of this machine and why?

Solution

Let

x_A = number of units of A produced

x_B = number of units of B produced

then the constraints are:

$12x_A + 25x_B <= 30(60)$ (assembly time)

$x_B >= 2(x_A/5)$

i.e. $x_B - 0.4x_A >= 0$

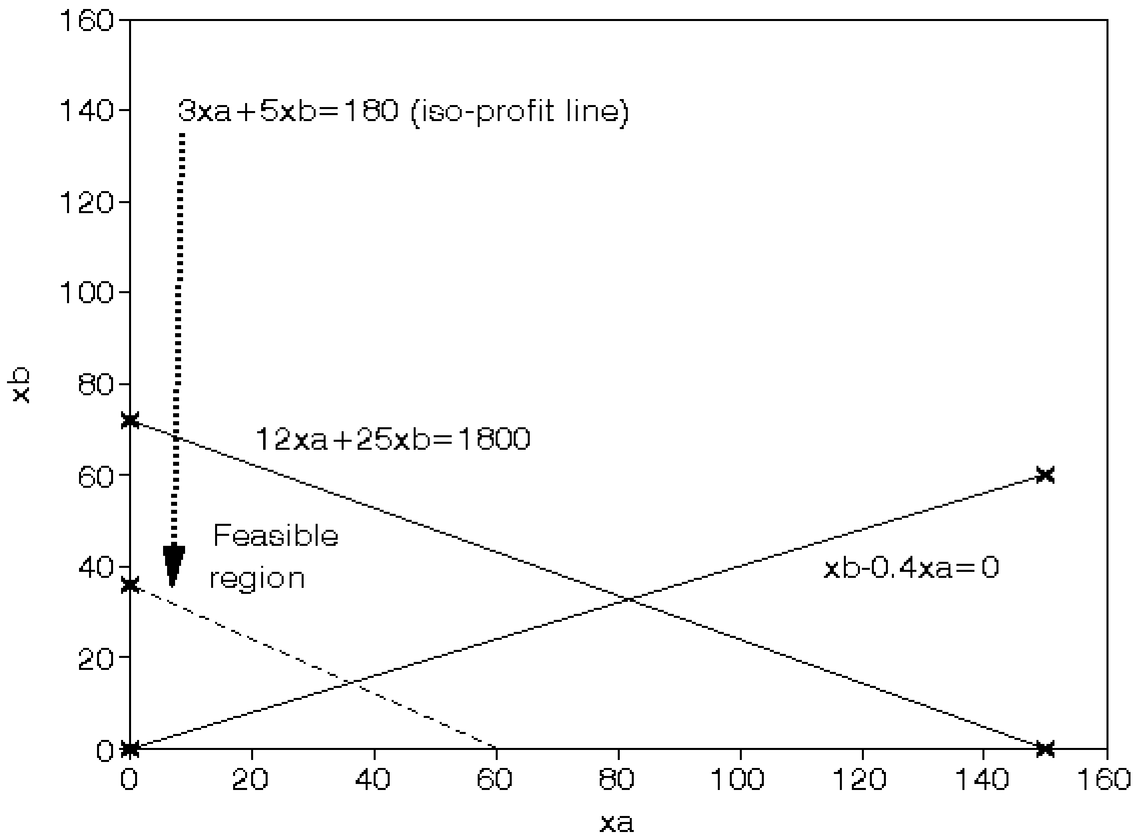
i.e. $5x_B >= 2x_A$ (technological)

where $x_A, x_B >= 0$

and the objective is

maximise $3x_A + 5x_B$

It is plain from the diagram below that the maximum occurs at the intersection of $12x_A + 25x_B = 1800$ and $x_B - 0.4x_A = 0$



Solving simultaneously, rather than by reading values off the graph, we have that:

x_A= (1800/22) = 81.8

x_B= 0.4x_A = 32.7

with the value of the objective function being £408.9

Doubling the assembly time available means that the assembly time constraint (currently $12x_A + 25x_B \le 1800$) becomes $12x_A + 25x_B \le 2(1800)$ This new constraint will be parallel to the existing assembly time constraint so that the new optimal solution will lie at the intersection of $12x_A + 25x_B = 3600$ and $x_B - 0.4x_A = 0$

i.e. at x_A = (3600/22) = 163.6

x_B= 0.4x_A = 65.4

with the value of the objective function being £817.8

Hence we have made an additional profit of £(817.8-408.9) = £408.9 and this is the *maximum* amount we would be prepared to pay for the hire of the machine for doubling the assembly time.

This is because if we pay more than this amount then we will reduce our maximum profit below the £408.9 we would have made without the new machine.

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3) Solve the following linear program:

maximise $5x_1 + 6x_2$

subject to

$x_1 + x_2 <= 10$

$x_1 - x_2 >= 3$

$5x_1 + 4x_2 <= 35$

$x_1 >= 0$

$x_2 >= 0$

Solution

It is plain from the diagram below that the maximum occurs at the intersection of

$5x_1 + 4x_2 = 35$ and

$x_1 - x_2 = 3$

Solving simultaneously, rather than by reading values off the graph, we have that

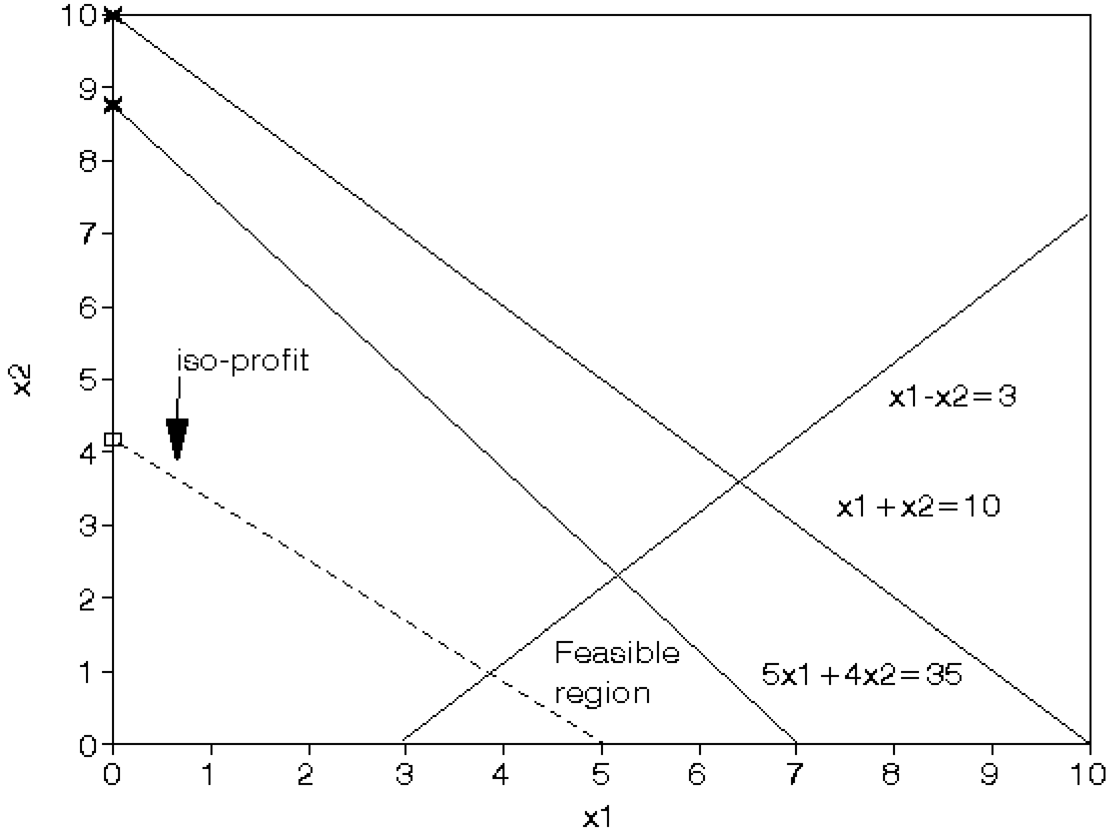
$5(3 + x_2) + 4x_2 = 35$

i.e. $15 + 9x_2 = 35$

i.e. $x_2 = (20/9) = 2.222$ and

$x_1 = 3 + x_2 = (47/9) = 5.222$

The maximum value is $5(47/9) + 6(20/9) = (355/9) = 39.444$



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4) A carpenter makes tables and chairs. Each table can be sold for a profit of £30 and each chair for a profit of £10. The carpenter can afford to spend up to 40 hours per week working and takes six hours to make a table and three hours to make a chair. Customer demand requires that he makes at least three times as many chairs as tables. Tables take up four times as much storage space as chairs and there is room for at most four tables each week.

Formulate this problem as a linear programming problem and solve it graphically.

Solution

Variables

Let

x_T = number of tables made per week

x_C = number of chairs made per week

Constraints

- total work time

$6x_T + 3x_C <= 40$

- customer demand

$x_C >= 3x_T$

- storage space

$(x_C/4) + x_T <= 4$

- all variables >= 0

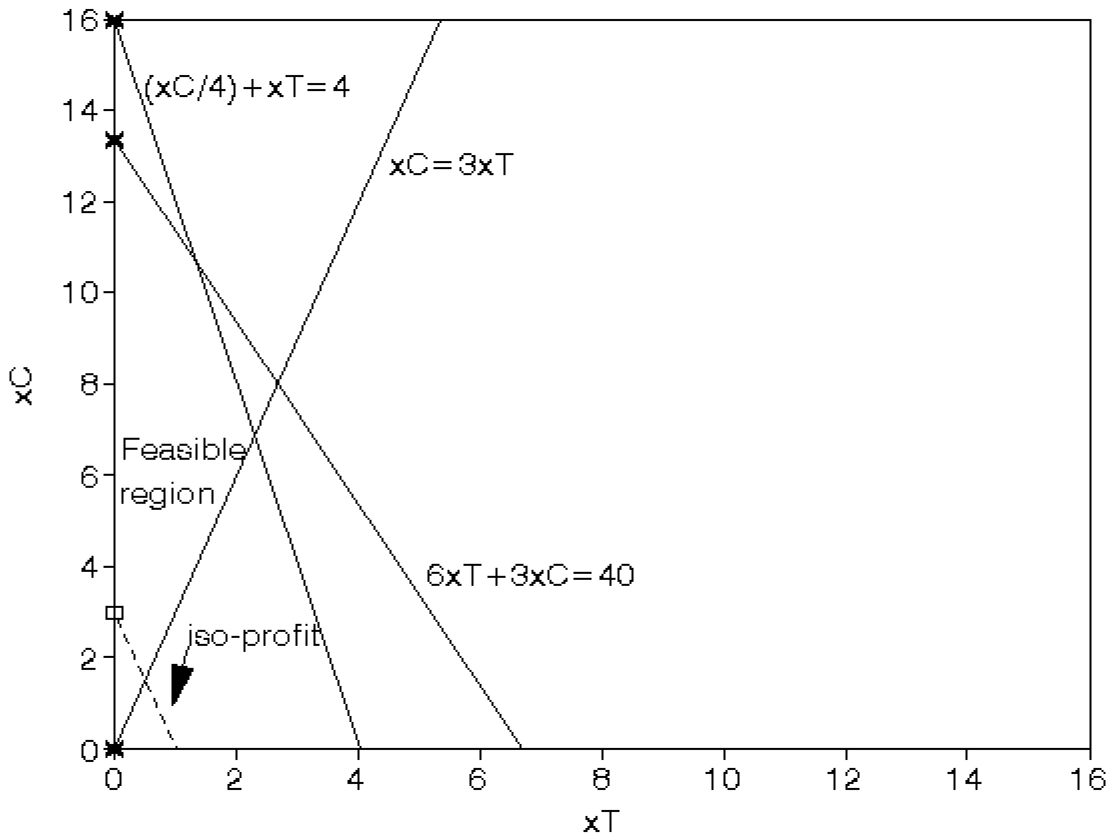
Objective

maximise $30x_T + 10x_C$

The graphical representation of the problem is given below and from that we have that the solution lies at the intersection of

$(x_C/4) + x_T = 4$ and $6x_T + 3x_C = 40$

Solving these two equations simultaneously we get $x_C = 10.667$, $x_T = 1.333$ and the corresponding profit = £146.667



Problems with no given solution

1. A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only \$1200 to spend and each acre of wheat costs \$200 to plant and each acre of rye costs \$100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is \$500 per acre of wheat and \$300 per acre of rye how many acres of each should be planted to maximize profits?

2. A gold processor has two sources of gold ore, source A and source B. In order to kep his plant running, at least three tons of ore must be processed each day. Ore from source A costs \$20 per ton to process, and ore from source B costs \$10 per ton to process. Costs must be kept to less than \$80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?

3. A publisher has orders for 600 copies of a certain text from San Francisco and 400 copies from Sacramento. The company has 700 copies in a warehouse in Novato and 800 copies in a warehouse in Lodi. It costs $5 to ship a text from Novato to San Francisco, but it costs $10 to ship it to Sacramento. It costs $15 to ship a text from Lodi to San Francisco, but it costs $4 to ship it from Lodi to Sacramento. How many copies should the company ship from each warehouse to San Francisco and Sacramento to fill the order at the least cost?